From Lorentz Force on Electron to Magnus Force on Vortex, Role of Experiments

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The vortex motion in a superfluid or a type II superconductor is similar to the electron motion in a magnetic field, because they both feel a transverse force. The vortex dynamics in a superconductor is a basic property of the superconductivity which remains controversial. It is also responsible for a large class of observed physical phenomena. We will examine this issue from the experimental point of view. In particular, we will compare the experiments which have set the stage to the Lorentz force and the experiments influencing our understanding of the Magnus force on vortices in superconductors.

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In hydrodynamics, we have learned a remarkable difference between the Newtonian dynamics and the Eulerian dynamics. A particle obeying Newton dynamics accelerates along the direction of applied force. A vortex in a flow field, however, always has a tendency to curve. If you push a vortex, it responds perpendicular to your push, if the background flow is at rest. If a vortex is at rest while there is a background flow, the vortex feels a force perpendicular to the direction of the flow. In daily life, we encounter numerous examples of this 'curving' nature of vortices.

For an inviscous hydrodynamic fluid, we have a well defined starting point, the Euler equation,

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{\nabla P}{\rho}. \tag{1}$$

Here ρ is the fluid mass density and P is the pressure. The above equation is applicable everywhere in space except at the singular point of the vortex core. The way we usually derive the force on a vortex [1] is to assume that there is a trapped foreign object, for instance a disc, in the vortex core and study the force on this object. The resulting Magnus force is given by

$$\mathbf{F} = \rho[(\mathbf{v}_0 - \mathbf{v}_v) \times \kappa]. \tag{2}$$

Here κ is the vorticity, $\mathbf{v}_v$ the velocity of the vortex and $\mathbf{v}_0$ the velocity of the background flow. For a steady state motion, the Magnus force force balances the applied external force. If there is no external force, the vortex always moves along the background flow. The force on a vortex is similar to the force on an electron in a magnetic field. If we define a fictitious magnetic field $\tilde{\mathbf{B}} = \rho\kappa$ and a fictitious electric field $\tilde{\mathbf{E}} = -\rho\mathbf{v}_0 \times \kappa$, the force on a vortex is identical to the force on an electron of a unit charge $\mathbf{F} = \tilde{\mathbf{E}} + \mathbf{v} \times \tilde{\mathbf{B}}$

In a neutral superfluid He^4, we can employ the two fluid model [2]. The Euler equation for the superfluid component is rather similar to that of hydrodynamic flow,

$$\frac{\partial \mathbf{v}_s}{\partial t} + (\mathbf{v}_s \cdot \nabla)\mathbf{v}_s = -\nabla\mu, \tag{3}$$

where μ is the chemical potential. The Magnus force also has a similar form,

$$\mathbf{F} = \rho_s[(\mathbf{v}_0 - \mathbf{v}_v) \times \kappa]. \tag{4}$$

Beside that the superfluid mass density ρ_s replaces the fluid mass density, there is also another important difference. The vorticity is quantized in a superfluid so that the only way to generate or annihilate a single vortex is to move it from(or to) the boundary.

For a charged superfluid, there are additional terms due to the coupling to the electromagnetic field in the Euler equation,

$$\frac{\partial \mathbf{v}_s}{\partial t} + (\mathbf{v}_s \cdot \nabla)\mathbf{v}_s = -\nabla\mu + \frac{e}{m}\left[\mathbf{E} + \frac{1}{c}(\mathbf{v}_s \times \mathbf{H})\right]. \tag{5}$$

Bearing in mind that the force is related to the difference in electro-chemical potential difference, we still obtain the same Magnus force as in the neutral superfluid case [3].

The type II superconductor is similar to a fermionic superfluid. In principle, we should be able to derive a hydrodynamic equation for the the Cooper pairs, which can be regarded as bosons. Such a hydrodynamic equation has been written down from the non-linear Schrödinger equation for a superconductor in the clean limit at low temperatures [4]. In general, we should expect that a hydrodynamic equation still exists, with a modified superfluid density and an effective Cooper pair mass which can be determined experimentally. Then we may ask why there is no consensus on the Magnus force in a superconductor [5] and where does the conflict of several different point of views originated.

The root of the disagreement is the experimental observation. Unlike in the hydrodynamic fluid motion where we have plenty of examples to demonstrate the Magnus force, in superconductors, we do not have such obvious examples. From the very beginning of the study of vortex dynamics in superconductors, we have associated the understanding of the Hall effect to the Magnus force.

Let us discuss in more detail the Hall effect in superconductors. For the time being let us assume that

the vortices in superconductors feel the Magnus force from the hydrodynamic equation and examine its consequences. Under this assumption, the equation of motion for a vortex takes the form of the Langevin equation similar to that of a charged particle in the presence of a magnetic field:

$$m_v \ddot{\mathbf{r}} = q_v \frac{\rho_s}{2} h \left(\mathbf{v}_s - \dot{\mathbf{r}} \right) \times \hat{z} - \eta \dot{\mathbf{r}} + \mathbf{F}_{pin} + \mathbf{f} \ , \qquad (6)$$

with an effective mass m_v, a pinning force $\mathbf{F}_{pin}$, a vortex viscosity η, and a fluctuating force $\mathbf{f}$. $q_v = \pm 1$ represent different vorticities. The viscosity is related to the fluctuating force by the usual fluctuation-dissipation theorem.

Now let us ignore the vortex interaction as well as the pinnings. Then we can solve the above equation to give

$$\dot{\mathbf{r}} = \frac{(\rho_s h/2)^2}{\eta^2 + (\rho_s h/2)^2} \mathbf{v}_s + q_v \frac{(\rho_s h/2)\eta}{\eta^2 + (\rho_s h/2)^2} \mathbf{v}_s \times \hat{z} \ . \quad (7)$$

According to the Josephson relation, the measured electric field $\mathbf{E}$ is given by

$$\mathbf{E} = -q_v \frac{h}{2e} n \, \mathbf{v}_l \times \hat{z} \ , \qquad (8)$$

and it can be rewritten as

$$\mathbf{E} = -\frac{1}{c} \mathbf{v}_l \times \mathbf{B} \ . \qquad (9)$$

We can calculate the longitudinal and Hall resistivity, ρ_{xx} and ρ_{xy} from the above equations by $\rho_{xx} = E_x/J$ and $\rho_{yx} = E_y/J$. The friction can be estimated by assuming that the vortex cores behave as normal electrons. Then we are ready to compare the calculated ρ_{xx} and ρ_{xy} with the experiments.

For the experiments which existed thirty years ago, the above results are already qualitatively different from the experimentally measured ones. The Hall angle, defined by $tan^{-1}\theta = \rho_{xy}/\rho_{xx}$ should be nearly 90 degree for the estimated one while it was only on the order of 0.5 degree for the data existing at that time. Naturally, the experimental facts led the theorists to find a way out. In their work, Bardeen and Stephen [6] argued that the Magnus force due to the hydrodynamic flow of the Cooper pairs does not exist in superconductor because the ionic background can contribute a term to cancel the vortex velocity dependent part of the Magnus force. Then the electrons at the nonsuperconducting vortex cores can contribute to a small Hall resistance in the same way as the normal electrons give rise to a Hall effect in an ordinary metal. The question seemed to have obtained a satisfactory answer for that time.

About thirty years later, the Magnus force in a type II superconductor has generated a renewed attention with the discovery of high-Tc superconductors especially after the observation of the so-called Hall anomaly. It was found that the Hall conductance changes sign in the

mixed states in many high-Tc superconductors and some conventional ones. Apparently it can not be explained by the Bardeen-Stephen model. Various attempts based on different ways to add another small contribution have been constructed. The contribution from the particle-hole asymmetry and the contribution of the back-flow are such examples. Overall speaking, the trend of the study of Magnus force on a superconductor was motivated and led by the experiments. The theories in this category have shown little prediction power.
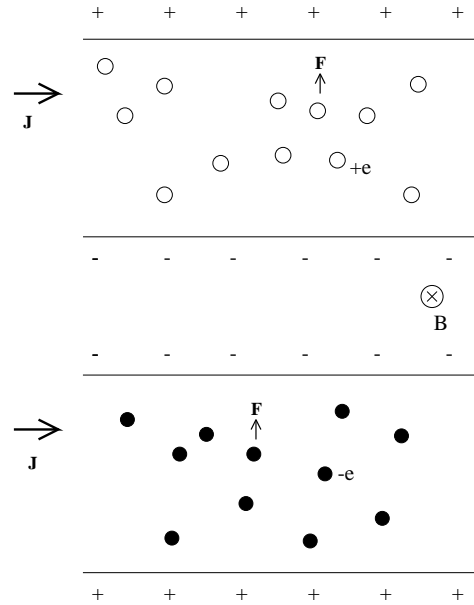

FIG. 1. When an electric current is passed through a metal bar in a magnetic field, the Hall voltage depends on the carrier type. The force this metal bar feels only depends on the total electric current carried through, not on how it is carried through. The upper graph shows the hole carriers and the lower graph shows the electron carriers.

The situation concerning the Lorentz force on an electron is very different. It might be helpful to examine the history on this undoubtedly successful sub-field of electron theory and see if we can learn anything from it. We have already shown that the vortex dynamics is similar to the electron dynamics in a magnetic field. What is the role that the Hall effect played in determining the Lorentz force and establishing the theory of electrons? We have to say little, if there is anything at all. Then what does this simple fact tell us?

Let us recall how we first introduce the Lorentz force in a text book? Take for example *The Feynman Lectures on Physics* [7]. It was introduced with two experiments. The first one measures the force on a wire carrying an electric current near a bar magnet, the second one the force between two wires carrying electric current simultaneously. In other words, the Lorentz force was introduced by genuine force measurement experiments. It was by these types of force measurements that we started to re-

alize there is an interaction between moving charges and the magnetic field. What would have happened if we did not have these type of experiments but only had the Hall effect? We probably would have taken a long detour toward understanding the Lorentz force and developing the theory of electrons.

There is a reason to believe the Hall effect in a type II superconductor has led the theorists to such a detour. The Hall effect in a superconductor is in many ways similar to that of a normal metal. In a normal metal, the electrons interact with the background lattice potential so that we have Bloch bands. They also interact with each other through the Pauli principle. As a result, the bandstructure determines the carrier types. The Hall coefficient can be positive or negative depending on the details. Although this is well known today, it was understood decades after the Lorentz force itself. If we did not know the Lorentz force beforehand, it would have been very difficult trying to understand the Lorentz force and the Hall effect at the same time. The vortices in a superconductor also feel background potentials. Pinnings from inhomogeneity inevitably exist. In addition, there is also an intrinsic pinning due to the interaction between the vortex and the lattice background. This coupling exists even when there are no defects in the sample. The vortices also interact with each other and they form a lattice at lower temperatures. When they form a lattice, the sliding of the whole lattice is only one of the modes of motion. Vortices can move by defect motion too. It has been pointed out that defect configurations act like different carrier types. Even when the temperature is higher than the melting temperature of the vortex lattice, the vortex-vortex interactions still cannot be ignored. We know perfectly well that many transport properties of a liquid are considerably influenced by the interactions.

In fact, without any arm-twisting modification of the basic superfluid hydrodynamics which governs the vortex motion, the Hall effect in a superconductor can also be explained in a similar way as the Hall effect in a normal metal. In such a framework, the Magnus force, like the Lorentz force is a basic property and is not sample dependent [8]. The vortex motion, however, is complicated by all kinds of possible defect motions. At low temperatures, the vacancy motion may dominate and give rise to the Hall anomaly [9].

Can we put the above framework into test experimentally? There is a difference between vortices and electrons concerning measurement. It is possible to study a single electron motion in vacuum so that it is free from any of the complications we face in a Hall effect measurement. However, for the superconductor, an individual vortex cannot penetrate through the sample. As long as the magnetic field is above H_{c1}, the vortices always form a lattice so that we have to consider vortex-vortex interaction. The situation is not hopeless, however. Still, much can be learned from the experiments we used to

introduce the Lorentz force. In our description of those experiments, we never considered the lattice background potential or the Fermi statistics. It was not even known at the time this type of experiments was first performed. Apparently those experiments are independent of carrier types. We will demonstrate more in fig.1. If we pass a current through a metal bar in a magnetic field, depending on the bandstructure of this metal, the Hall effect may have different signs. However, the force on this metal bar is given by the total Lorentz force $\mathbf{J} \times \mathbf{B}$ and is independent of the details on how the electrons are carried from one end of the bar to the other.

This simple experimental construction tells us an important way to construct an experiment to measure the Magnus force on the vortex. If we can follow the same principle to construct a direct force measurement when the vortices plays the role of electrons, we should be able to determine the Magnus force even we do not know how the vortices actually move, by defect motion, plastic flow or other ways.
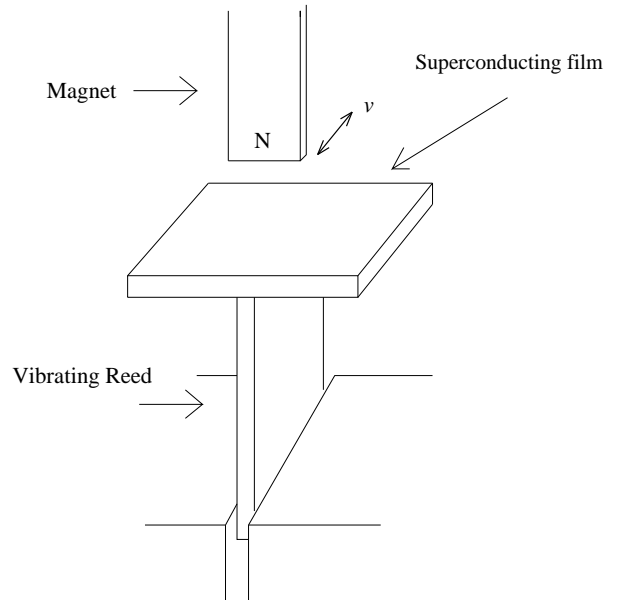

FIG. 2. Schematic drawing of the experimental setup used to observe the Magnus force on the vortices in a superconductor.

In principle we can construct a experiment almost exactly as the first experiment shown in the textbook [7]. We can attach two wires to a superconductor, place it in a magnetic field and pass a current through. Then we can measure the force on the superconductor. We also need to determine the direction and the magnitude of the vortex current. This is not a problem if we can measure the longitudinal and Hall resistance at the same time. There is only one drawback that all the high Tc superconductors are not wire-friendly. It is not easy to attach and manage so many wires to a high Tc superconductor and perform a force measurement at the same time.

A direct force experiment was carried out recently with a slightly different design. The vortices were driven into motion by a small magnet which was vibrating above a superconducting film. When the magnet is moving at a low speed, we can neglect the vortex mass and assume the total force on each vortex to balance. On each vortex, there is a force from the moving magnet, a transverse force acting on the vortex from the superconductor (the Magnus force), a pinning force and a friction force from the underlying lattice and the interaction force from other vortices. Now let us examine the total force summed over all vortices. The total force of vortex interactions vanishes. In the direction parallel to the motion of the magnet, the total force from the moving magnet to the vortices will balance the total pinning and the total friction from the superconductor. In the transverse direction, the total force from the moving magnet to the vortices will balance the total transverse force on vortices from the superconductor, i.e. the total velocity dependent part of the Magnus force. Thus in the transverse direction, a reaction force to the total Magnus force which is coming from the moving magnet to the vortices will be passed entirely to the superconductor. This is exactly the force the superconducting film feels. To measure this force on the film, the film is mounted on a vibrating reed and the frequency of the vibration of the magnet is tuned to sweep through the resonance frequency of the vibrating reed. To maximize the signal, the direction of the vibration of the reed is adjusted to be perpendicular to the motion of the magnet.

Indeed this force measurement has provided us with something valuable. The Magnus force was found to have the same sign and the order of magnitude as predicted from a hydrodynamic equation. Although more experiments are still needed to complete this subject, at least we started to have a new direction to design and to understand our experiments. The Hall effect is a very interesting subject by itself. However, it did not help the development of the theory of electrons and it most likely will not help developing the theory of vortex dynamics in superconductors. We can put the Hall effect aside when we try to derive the equation of motion for the vortex. We only need to study the theory from a theoretical point of view. Hopefully, something new can come out of it.

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